Muon Physics: A Pillar of the Standard Model

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Since its discovery in the 1930s, the muon has played an important role in our quest to understand the sub-atomic theory of matter. The muon was the first second-generation standard-model particle to be discovered, and its decay has provided information on the $(Vector - Axial\,Vector)$ structure of the weak interaction, the strength of the weak interaction, G_F, and the conservation of lepton number (flavor) in muon decay. The muon's anomalous magnetic moment has played an important role in restricting theories of physics beyond the standard standard model, where at present there is a 3.4 σ difference between the experiment and standard-model theory. Its capture on the atomic nucleus has provided valuable information on the modification of the weak current by the strong interaction which is complementary to that obtained from nuclear β decay.

KEYWORDS: muon, weak decay, muon capture, magnetic moment, lepton flavor violation

1. Introduction

The muon was first observed in a Wilson cloud chamber by Kunze[1] in 1933, where it was reported to be "a particle of uncertain nature." In 1936 Anderson and Neddermeyer[2] reported the presence of "particles less massive than protons but more penetrating than electrons" in cosmic rays, which was confirmed in 1937 by Street and Stevenson,[3] Nishina, Tekeuchi and Ichimiya,[4] and by Crussard and Leprince-Ringuet.[5] The Yukawa theory of the nuclear force had predicted such a particle, but this "mesotron" as it was called, interacted too weakly with matter to be the carrier of the strong force. Today we understand that the muon is a second generation lepton, with a mass about 207 times the electron's. Like the electron, the muon obeys quantum electrodynamics, and can interact with other particles through the electromagnetic and weak forces. Unlike the electron which appears to be stable, the muon decays through the weak force.

The muon lifetime of 2.2 μs permits one to make precision measurements of its properties, and to use it as a tool to study the semileptonic weak interaction, nuclear properties, as well as magnetic properties of condensed matter systems. The high precision to which the muonium (μ^+e^- atom) hyperfine structure can be measured and calculated makes it a significant input parameter in the determination of fundamental constants.[6] In this review, I will focus on the role of the muon in particle physics.

A beam of negative muons can be brought to rest in matter, where hydrogen-like atoms are formed, with a nuclear charge of Z. The Bohr radius for a hydrogen-like atom is inversely proportional to the orbiting particle's mass ($r_n = [n^2\hbar c]/[mc^2 Z\alpha]$), so that for the lowest quantum numbers of high-Z muonic atoms, the muon is well inside of the atomic electron cloud, with the Bohr radius of the $1S$ atomic state well inside the nucleus. The $2P \rightarrow 1S$ x-ray energies are shifted because of the modification of the Coulomb potential inside the nucleus, and these x rays have provided information on nuclear root-mean-square charge radii. The Lamb shift in muonic hydrogen, ΔE_{2P-2S}, which is being measured at at the Paul Scherrer Insitut (PSI), is given by[7] $\left\{209.974(6) - 5.226R_p^2 + 0.036R_p^3\right\}$ meV, where R_p is the proton rms charge radius. This experiment should provide a precise measurement of R_p. The weak nuclear capture, called ordinary muon capture (OMC), of the muon on the atomic nucleus following the cascade to the $1S$ ground state, $\mu^- +_Z \mathcal{N} \rightarrow_{Z-1} \mathcal{N} + \nu_\mu$, is the analog to the weak capture of a K-shell electron by the nucleus, and provides information on the modification of the weak interaction by the hadronic matter.

The muon mass of ~ 106 MeV restricts the muon to decay into the electron, neutrinos, and photons. Thus muon decay is a purely leptonic process, and the dominant decay mode is $\mu^- \rightarrow e^- + \nu_\mu + \bar{\nu}_e$. This three-body decay tells us that the individual lepton number, electron and muon, is conserved separately, and that the two flavors (kinds) of neutrinos are distinct particles.[10] Here the μ^- and e^- are "particles" and the μ^+ and e^+ are the antiparticles. In the 1950s, it became possible to make pions, and thus muons, in the laboratory. The energetically favorable decay $\mu^+ \rightarrow e\gamma$ was searched for and not found[8] to a relative branching ratio of $< 2 \times 10^{-5}$. Also searched for was the neutrinoless capture of a μ^- on an atomic nucleus, $\mu^- + \mathcal{N} \rightarrow e^- + \mathcal{N}$, which was not found at the level of $\sim 5 \times 10^{-4}$. Such processes are said to "violate lepton flavor," and continue to be the object of present and planned studies reaching to sensitivities of 10^{-14} and 10^{-16}, respectively.

The muon, like the electron, is a spin 1/2 lepton, with a magnetic moment given by

$$\vec{\mu}_s = g_s(\frac{q}{2m})\vec{s}; \qquad \mu = (1+a)\frac{q\hbar}{2m}; \qquad a \equiv \frac{(g_s - 2)}{2};$$

$$(1)$$

where the muon charge $q = \pm e$, and g_s, the Landé g-

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factor is slightly greater than the Dirac value of 2. The middle equation above is useful from a theoretical point of view, as it separates the magnetic moment into two pieces: the Dirac moment which is unity in units of the appropriate magneton, $e\hbar/2m$, and is predicted by the Dirac equation; and the anomalous (Pauli) moment, where the dimensionless quantity a is referred to as *the anomaly*. The muon anomaly, like the electron's, arises from radiative corrections that are discussed below.

When the muon was discovered, it was an unexpected surprise. Looking at this from our 21st century perspective, it is easy to forget how we reached what is now called the "standard model" of subatomic physics, which incorporates three generations of leptons, e, μ and τ and their neutrinos; three generations of quarks; the electro-weak gauge bosons, γ W and Z; and the gluons that carry the strong force. When this author joined the field as a graduate student in the mid 1960s, none of this was clear. Quarks were viewed by many as a mathematical device, not as constituent particles. Even after quarks were inferred from deep inelastic electron scattering off the proton, we only knew of the existence of three of them. While the $V - A$ structure of the weak interaction was first inferred from nuclear β decay, the study of muon decay has provided a useful laboratory in which to study the purely leptonic weak interaction, to search for physics beyond the standard model, such as additional terms in the interaction besides the standard-model $V - A$ structure, as well as looking for standard model forbidden decays like $\mu \to e\gamma$. For many years, the experimental value of the muon's anomalous magnetic moment has served to constrain physics beyond the standard model, and continues that role today.

2. Muon Decay and G_F

The muon decay $\mu^- \to e^- \nu_\mu \bar{\nu}_e$ is purely leptonic. Since $m_\mu << M_W$, muon decay can be described by a local four-fermion (contact) interaction. While nonrenormalizeable, at low energies it provides an excellent approximation to the full electroweak theory. The weak Lagrangian is written as a current-current interaction, where the leptonic current is of the $(V - A)$ form, $\bar{u}\gamma_\lambda(1 - \gamma_5)u$.

Michel[11] first wrote down a parameterization of muon decay, defining five parameters, ρ, η, ξ, δ and h, which are combinations of the different possible couplings allowed by Lorentz invariance in muon decay. The standard model has clear predictions for these parameters and they have been measured repeatedly over the intervening years to search for physics beyond the standard model. This tradition continues today, with the TWIST experiment at TRIUMF, which is mid-way through a program to improve on the precision of the Michel parameters by an order of magnitude.[12] While there are some scenarios in which new physics would conspire to leave the Michel parameters at their standard model value,[13] a variance from the standard-model values would be a clear sign of new physics at work.

The muon lifetime, see Fig. 1 is directly related to the strength of the weak interaction, which in Fermi theory is described by the constant G_F. The standard-model
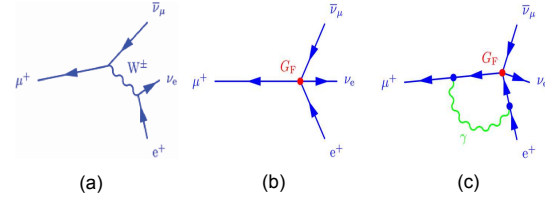

Fig. 1. Muon decay in (a) the standard model; (b) the Fermi theory; (c) radiative corrections to the Fermi theory.

electroweak gauge coupling g is related to G_F by[14]

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2}(1 + \Delta r) \qquad (2)$$

where Δr represents the weak boson mediated tree-level process and its radiative corrections.[15] In the standard model, the Fermi constant is related to the vacuum expectation value of the Higgs field by $G_F = 1/(\sqrt{2} \, v^2)$.

While the Fermi theory is nonrenormalizeable, the QED radiative corrections are finite to first order in G_F, and to all orders in the fine-structure constant α. This gives the relationship[14] between G_F and the muon lifetime, τ_μ,

$$\frac{1}{\tau_\mu} = \frac{G_F^2 m\mu^5}{192\pi^3}(1 + \Delta q) \qquad (3)$$

where Δq is the sum of phase space, and QED and hadronic radiative corrections. More properly one should write G_μ since new physics contributions could make G different for the three leptons.[16]

The MuLan experiment at PSI has recently reported a new measurement of the muon lifetime $2.197\,013(21)(11)$ μs (± 11 parts per million (ppm)),[17] to be compared with the previous world average $2.197\,03(4)$ μs (19 ppm).[18] The new world average muon lifetime of $2.197\,019(21)$ μs gives, assuming only standard-model physics in muon decay, $G_F = 1.166\,371(6) \times 10^{-5}$ GeV^2 (5 ppm). This experiment should eventually reach a precision of 1 ppm on τ_μ.

3. Nuclear Muon Capture

The weak capture of a muon on a proton has much in common with nuclear β decay. As for other low-energy weak processes, the interaction can be described as a current-current interaction with the $(V - A)$ leptonic current given by $\bar{u}_{\nu_\mu}\gamma^\lambda(1 - \gamma_5)u_\mu$. Because the strong interaction can induce additional couplings,[19] the hadronic current is more complicated. The most general form of the vector current allowed by Lorentz invariance is[20]

$$\bar{u}_n(p') \left[g_V(q^2)\gamma^\lambda + i\frac{g_M(q^2)}{2m_N}\sigma^{\lambda\nu}q_\nu + \frac{g_S(q^2)}{m_\mu}q^\lambda \right] u_p(p). \qquad (4)$$

The corresponding form of the axial-vector current is

$$\bar{u}_n(p') \left[-g_A(q^2)\gamma^\lambda\gamma_5 - \frac{g_P(q^2)}{m_\mu}\gamma_5 q^\lambda - i\frac{g_T(q^2)}{2m_N}\sigma^{\lambda\nu}q_\nu\gamma_5 \right] u_p(p), \qquad (5)$$

where m_μ and m_N are the muon and nucleon masses respectively; the $g(q^2)$ are the induced form factors: vector, weak magnetism, scalar, axial-vector, pseudoscalar and

tensor. The scalar and tensor terms are called "second class currents" because of their transformation properties under G-parity, and in the standard model are expected to be quite small.[19] It is traditional to set these second-class currents equal to zero.

Nuclear β decay is sensitive to the vector, axial-vector and weak-magnetism form factors, but in muon capture the the capture rate has a measurable contribution from the induced pseudoscalar interaction, the least well known of the weak nucleon form factors. Radiative muon capture (RMC), $\mu^- + p \to n + \gamma + \nu_\mu$, should in principle be more sensitive to the induced pseudoscalar coupling than OMC, since with the three-body final state, q^2 can get closer to the pion pole than is possible in ordinary muon capture, which was pointed out many years ago.[21, 22] The interested reader is referred to the review by Gorringe and Fearing for further discussion.[19]

In the past, current algebra and the Goldberger-Trieman relation expressed g_P in terms of g_A. With the development of quantum chromodynamics (QCD), and chiral perturbation theory, new interest has developed in the value of g_P.[19] The presently accepted theory value is $g_P(q^2 = -0.88m_\mu^2) = 8.26 \pm 0.23$.[19]

The experimental history is rather interesting. For many years the radiative capture reaction $\mu^- + p \to n + \gamma + \nu_\mu$ was thought to be the "golden" channel to study. However, this experiment is extremely difficult, and it was first observed experimentally in the 1990s.[23, 24] To achieve an adequate muon stopping rate, it was necessary to stop the muons a liquid hydrogen target. The value obtained for g_p by this experiment was in disagreement with both the ordinary muon capture experiment in liquid hydrogen, and with the theoretical expectation.

The muon chemistry in hydrogen, especially liquid, is rather complicated, and may be the source of these discrepancies. The incident μ^- first forms an atom with a proton in the hydrogen target, but then quickly picks up a second proton to form a $p\mu p$ molecule with the protons in the ortho $(J = 1)$ state. The ortho state of this molecule can decay to the $J = 0$ (para) state. The ortho and para proton states have different admixtures of the muon-proton spin: ortho has 3/4 singlet and 1/4 triplet $\mu - p$ and para has 1/4 singlet and 3/4 triplet. Since the muon capture rate is 40 times greater from the singlet $\mu^- p$ state than from the triplet, it is essential that Λ_{OP}, the ortho to para transition rate in the $p\mu p$ molecule, is known in order to extract g_p from the measured capture rate in liquid hydrogen.[19] Even after a recent measurement[25] of the transition rate Λ_{OP}, the difficulty in accommodating previous results on ordinary and radiative muon capture results in hydrogen continues. The complications of muon chemistry in liquid hydrogen can be avoided by using a 10 bar ultra-pure hydrogen target, which has a density 1.16% that of liquid hydrogen. At this lower density, the sensitivity to Λ_{OP} is greatly reduced. It is this approach that the recent MuCap experiment at PSI has used.

The MuCap experiment stops μ^- in a gaseous hydrogen target that functions as a time projection chamber (TPC), making it possible to determine where the muon stops in the target. A comparison of the μ^- life-time in this protonium target to the the free μ^+ lifetime, gives the capture rate and determines g_p. The MuCap experiment has recently reported a first result,[26] $g_p(q^2 = -0.88m_\mu^2) = 7.3 \pm 1.1$, consistent with the expectation from chiral perturbation theory. They have a factor of four more data which are being analyzed. While it is not clear what is wrong with the previous ordinary muon capture and radiative capture experiments, the MuCap result seems to indicate that a modern experiment, with a gaseous target and information from the TPC, has settled the long-standing discrepancy.

4. The Magnetic and Electric Dipole Moments

The electric and magnetic dipole moments have been an integral part of relativistic electron (lepton) theory since Dirac's famous 1928 paper, where he pointed out that an electron in external electric and magnetic fields has "the two extra terms

$$\frac{eh}{c}(\sigma, \mathbf{H}) + i\frac{eh}{c}\rho_1(\sigma, \mathbf{E}), \qquad (6)$$

...when divided by the factor $2m$ can be regarded as the additional potential energy of the electron due to its new degree of freedom.[27]" These terms represent the magnetic dipole (Dirac) moment and electric dipole moment interactions with the external magnetic and electric fields.

In modern notation, the magnetic dipole moment (MDM) interaction becomes

$$\bar{u}_\mu \left[eF_1(q^2)\gamma_\beta + \frac{ie}{2m_\mu}F_2(q^2)\sigma_{\beta\delta}q^\delta \right] u_\mu \qquad (7)$$

where $F_1(0) = 1$, and $F_2(0) = a_\mu$. The electric dipole moment (EDM) interaction is

$$\bar{u}_\mu \left[\frac{ie}{2m_\mu}F_2(q^2) - F_3(q^2)\gamma_5 \right] \sigma_{\beta\delta}q^\nu u_\mu \qquad (8)$$

where $F_2(0) = a_\mu$, $F_3(0) = d_\mu$, with

$$d_\mu = \left(\frac{\eta}{2}\right)\left(\frac{e\hbar}{2mc}\right) \simeq \eta \times 4.7 \times 10^{-14} \, e \, \text{cm}. \qquad (9)$$

(This η, which is the EDM analogy to g for the MDM, should not be confused with the Michel parameter η.)

The existence of an EDM implies that both P and T are violated.[28, 29] This can be seen by considering the non-relativistic Hamiltonian for a spin one-half particle in the presence of both an electric and magnetic field: $\mathcal{H} = -\vec{\mu} \cdot \vec{B} - \vec{d} \cdot \vec{E}$. The transformation properties of $\vec{E}$, $\vec{B}$, $\vec{\mu}$ and $\vec{d}$ are given in the Table I, and we see that while $\vec{\mu} \cdot \vec{B}$ is even under all three, $\vec{d} \cdot \vec{E}$ is odd under both P and T. While parity violation has been observed in many weak processes, direct T violation has only been observed in the neutral kaon system.[30] In the context of CPT symmetry, an EDM implies CP violation, which is allowed by the standard model for decays in the neutral kaon and B-meson sectors.

Observation of a non-zero electron or muon EDM would be a clear signal for new physics. To date no permanent EDM has been observed for the electron, the neutron, or an atomic nucleus, with the experimental limits given in Table II. It is interesting to note that

Table I. Transformation properties of the magnetic and electric fields and dipole moments.

	$\vec{E}$	$\vec{B}$	$\vec{\mu}$ or $\vec{d}$
P	-	+	+
C	-	-	-
T	+	-	-

in his original paper[27] Dirac stated "The electric moment, being a pure imaginary, we should not expect to appear in the model. It is doubtful whether the electric moment has any physical meaning, since the Hamiltonian ...that we started from is real, and the imaginary part only appeared when we multiplied it up in an artificial way in order to make it resemble the Hamiltonian of previous theories." Even in the 4th edition of his quantum mechanics book from 1958, well after the suggestion of Purcell and Ramsey[31] that one should search for a permanent EDM, Dirac held fast to this point of view.

While CP violation is widely invoked to explain the baryon-antibaryon asymmetry of the universe, the CP violation observed to date in the neutral kaon, and in the B meson sectors is too small to explain it. This CP deficit has motived a broad program of searches for EDMs in a range of systems. Many extensions to the standard model, such as supersymmetry, do not forbid new sources of CP-violation, and the failure to observe it has placed severe restrictions on many models.

Table II. Measured limits on electric dipole moments, and their standard model values

Particle	Present EDM Limit (e cm)	Standard Model Value (e cm)
n	2.9×10^{-26} (90%CL)[32]	10^{-31}
e^-	$\sim 1.6 \times 10^{-27}$ (90%CL)[33]	10^{-38}
μ	$< 10^{-18}$ (CERN)[34]	10^{-35}
	$\sim 10^{-19}$ † (E821)	
^{199}Hg	2.1×10^{-28} (95%CL)[35]	

†Estimated

The magnetic dipole moment can differ from its Dirac value ($g = 2$) for several reasons. Recall that the proton's g-value is 5.6 ($a_p = 1.79$), a manifestation of its quark-gluon internal structure. On the other hand, the leptons appear to have no internal structure, and the MDMs are thought to arise from radiative corrections, i.e. from virtual particles that couple to the lepton. We would emphasize that these radiative corrections need not be limited to the standard-model particles, but rather the physical values of the lepton anomalies represent a sum-rule over *all* virtual particles in nature that can couple to the lepton, or to the photon through vacuum polarization loops.

The standard model value of a lepton's anomaly, a_ℓ, has contributions from three different sets of radiative processes: quantum electrodynamics (QED)– with loops containing leptons (e, μ, τ) and photons; hadronic – with hadrons in vacuum polarization loops; and weak – with loops involving the bosons W, Z, and Higgs. Examples

are shown in Fig. 2. Thus

$$a_{e,\mu}^{(SM)} = a_{e,\mu}^{(QED)} + a_{e,\mu}^{(hadronic)} + a_{e,\mu}^{(weak)}. \qquad (10)$$

The dominant contribution from quantum electrodynamics (QED), called the Schwinger term,[36] $a^{(2)} = \alpha/2\pi$, and is shown diagrammatically in Fig. 2(a). The QED contributions have been calculated through four loops, with the leading five-loop contributions calculated.[37] Examples of the hadronic and weak contributions are given in Fig. 2(b)-(e).

The hadronic contribution cannot be calculated directly from QCD, since the energy scale is very low ($m_\mu c^2$), although Blum has performed a proof of principle calculation on the lattice.[44] Fortunately dispersion theory[38] gives a relationship between the vacuum polarization loop and the cross section for $e^+e^- \to$ hadrons,

$$a_\mu(\text{Had}; 1) = (\frac{\alpha m_\mu}{3\pi})^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s) R(s); \qquad (11)$$

$R \equiv \{\sigma_{tot}(e^+e^- \to \text{hadrons})\} / \{\sigma_{tot}(e^+e^- \to \mu^+\mu^-)\}$, and experimental data are used as input[38, 39]
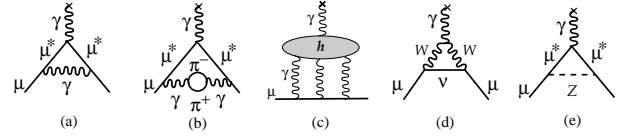

Fig. 2. The Feynman graphs for: (a) Lowest-order QED (Schwinger) term; (b)Lowest-order hadronic contribution; (c) hadronic light-by-light contribution; (d)-(e) the lowest order electroweak W and Z contributions. The * emphasizes that in the loop the muon is off-shell. With the present limits on m_h, the contribution from the single Higgs loop is negligible.

The muon anomaly is sensitive to a number of potential candidates for physics beyond the standard model:[41]

(1) muon substructure, where the contribution depends on the substructure scale Λ as

$$\delta a_\mu(\Lambda_\mu) \simeq \frac{m_\mu^2}{\Lambda_\mu^2}, \qquad (12)$$

(2) W-boson substructure.

(3) new particles that couple to the muon, such as the supersymmetric partners of the weak gauge bosons,

(4) extra dimensions

The potential contribution from supersymmetry has generated a lot of attention,[42, 43] the relevant diagrams are shown in Fig. 3 below. A simple model with equal masses[41] gives

$$a_\mu^{(SUSY)} \simeq \frac{\alpha(M_Z)}{8\pi \sin^2 \theta_W} \frac{m_\mu^2}{\tilde{m}^2} \tan\beta \left(1 - \frac{4\alpha}{\pi} \ln \frac{\tilde{m}}{m_\mu}\right) \quad (13)$$

$$\simeq (\text{sgn}\mu) \, 13 \times 10^{-10} \, \tan\beta \left(\frac{100 \text{ GeV}}{\tilde{m}}\right)^2 \qquad (14)$$

where $\tan\beta$ is the ratio of the two vacuum expectation values of the two Higgs fields. If the SUSY mass scale were known, then $a_\mu^{(SUSY)}$ would provide a clean way to determine $\tan\beta$.

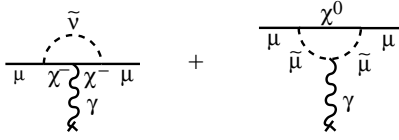Fig. 3. The lowest-order supersymmetric contributions to the muon anomaly. The χ are the superpartners of the standard-model gauge bosons.

4.1 Measurement of the Anomalous Magnetic Dipole Moment

Measurement of the magnetic anomaly uses the spin motion in a magnetic field. For a muon moving in a magnetic field, the spin and momentum rotate with the frequencies:

$$\vec{\omega}_S = -\frac{qg\vec{B}}{2m} - \frac{q\vec{B}}{\gamma m}(1-\gamma); \quad \vec{\omega}_C = -\frac{q\vec{B}}{m\gamma}. \qquad (15)$$

The spin precession *relative to the momentum* occurs at the difference frequency, ω_a, between the spin and cyclotron frequencies, Equation 15,

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_C = -\left(\frac{g-2}{2}\right)\frac{q\vec{B}}{m} = -a_\mu\frac{q\vec{B}}{m}. \qquad (16)$$

The magnetic field in Eq. 16 is the average field seen by the ensemble of muons. This technique has been used in all but the first experiments by Garwin, et al.,[45] which used stopping muons, to measure the anomaly. After Garwin, et al., made a 12% measurement of the anomaly, a series of three beautiful experiments at CERN culminated with a 7.3 ppm measure of a_μ.[34]

In the third CERN experiment, a new technique was developed based on the observation that electrostatic quadrupoles could be used for vertical focusing. With the velocity transverse to the magnetic field ($\vec{\beta} \cdot \vec{B} = 0$), the spin precession formula becomes

$$\vec{\omega}_a = -\frac{q}{m}\left[a_\mu\vec{B} - \left(a_\mu - \frac{1}{\gamma^2-1}\right)\frac{\vec{\beta}\times\vec{E}}{c}\right]. \qquad (17)$$

For $\gamma_{\text{magic}} = 29.3$, ($p_{magic} = 3.09$ GeV/c), the second term vanishes; one is left with the simpler result of Equation 16, hence the name "magic", and the electric field does not contribute to the spin precession relative to the momentum. There are two major advantages of using the magic γ and a uniform magnetic field: (i) the knowledge needed on the muon trajectories to determine the average magnetic field is much less than when gradients are present, and the more uniform field permits NMR techniques to realize their full accuracy, thus increasing the knowledge of the B-field. The spin precession is determined almost completely by Eq. 16, which is independent of muon momentum; *all* muons precess at the same rate. This technique was used also in experiment E821[40] at the Brookhaven National Laboratory Alternating Gradient Synchrotron (AGS). The reader is referred to Ref. [38] for a discussion of muon decay relevant to E821, and to Ref. [40] for details of E821.

Muons are stored in a storage ring,[40] and the arrival time and energy of the decay electrons is measured.

When a single energy threshold is placed on the decay electrons, the number of high-energy electrons is modulated by the spin precession frequency, Eq 17, producing the time distribution

$$N(t, E_{th}) = N_0(E_{th})e^{\frac{-t}{\gamma\tau}}[1 + A(E_{th})\cos(\omega_a t + \phi(E_{th}))]. \qquad (18)$$

as shown in Fig. 4.[40] The value of ω_a is obtained from a least-squares fit to these data. The five-parameter function (Eq. 18) is used as a starting point, but many additional small effects must be taken into account.[40]
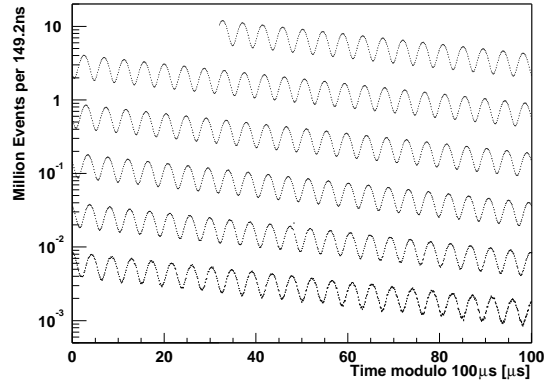

Fig. 4. The time spectrum of 3.6×10^9 electrons with energy greater than 1.8 GeV from the 2001 data set. The diagonal "wiggles" are displayed modulo 100 μs.

In E821, both μ^+ and μ^- were measured, and assuming CPT invariance, the final result obtained by E821,[40] $a_\mu^{\text{exp}} = 116\,592\,080\,(63) \times 10^{-11}$, is shown in Fig. 5, along with the individual measurements and the standard-model value. The present standard-model value is[38] $a_\mu^{\text{SM}(06)} = 116\,591\,785\,(61) \times 10^{-11}$, and one finds $\Delta a_\mu = 295(88) \times 10^{-11}$, a 3.4 σ difference.
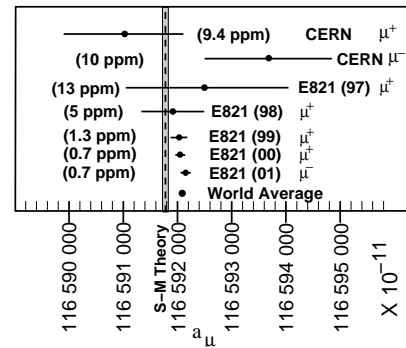

Fig. 5. Measurements of the muon anomaly, indicating the value, as well as the muon's sign. As indicated in the text, to obtain the value of a_{μ^-} and the world average CPT invariance is assumed. The theory value is taken from Ref. [38], which uses electron-positron annihilation to determine the hadronic contribution.

One candidate for the cosmic dark matter is the lightest supersymmetric partner, the neutralino, χ^0 in Fig. 3.

In the context of a constrained minimal supersymmetric model (CMSSM), $(g-2)_\mu$ provides an orthogonal constraint on dark matter[46] from that provided by the WMAP survey, as can be seen in Fig. 6.
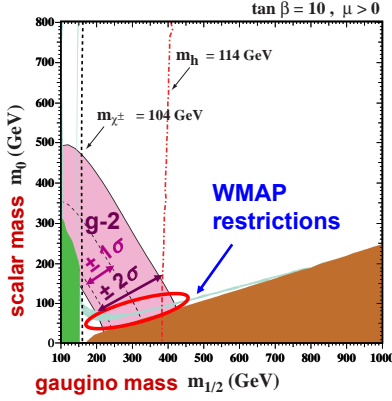

Fig. 6. Limits on dark matter placed by various inputs in CMSSM, with $\tan\beta = 10$. The Δ between experiment and standard-model theory is from Ref. [38], see text. The brown wedge on the lower right is excluded by the requirement the dark matter be neutral. Direct limits on the Higgs and chargino $\chi^\pm$ masses are indicated by vertical lines. Restrictions from the WMAP satellite data are shown as a light-blue line. The $(g-2)$ 1 and 2-standard deviation boundaries are shown in purple. The region "allowed" by WMAP and $(g-2)$ is indicated by the ellipse, which is further restricted by the limit on M_h. (Figure courtesy of K. Olive)

With the apparent $3.4\,\sigma$ difference between theory and experiment, a new experiment to improve the error by a factor of 2 to 2.5 has been proposed to Brookhaven laboratory, but at present it is not funded. The theoretical value will continue to be improved, both with the expected availability of additional data on e^+e^- annihilation to hadrons, and with additional work on the hadronic light-by-light contribution.[38]

4.2   The Search for a Muon Electric Dipole Moment

With an EDM present, the spin precession frequency relative to the momentum must be modified. The total frequency becomes $\vec{\omega} = \vec{\omega}_a + \vec{\omega}_\eta$, where

$$\vec{\omega}_\eta = -\frac{q}{m}\left[\frac{\eta}{2}\left(\frac{\vec{E}}{c} + \vec{\beta}\times\vec{B}\right)\right], \qquad (19)$$

with η defined by Eq. 9, and ω_a by Eq. 17. The spin motion resulting from the motional electric field, $\vec{\beta}\times\vec{B}$ is the dominant effect, so ω_η is transverse to $\vec{B}$. An EDM would have two effects on the precession, there would be a slight tipping of the precession plane, which would cause a vertical oscillation of the centroid of the decay electrons that out of phase with the ω_a precession; and the observed frequency ω would be larger,

$$\omega = \sqrt{\omega_a^2 + \left(\frac{q\eta\beta B}{2m}\right)^2}. \qquad (20)$$

The muon limit in Table II, placed by the non-observation of the vertical oscillation, is dominated by

systematic effects. The limit obtained by this method in the CERN experiment,[34] and likely to be obtained by E821, cannot directly exclude the possibility that the entire difference between the measured and standard-model values of a_μ could be caused by a muon EDM. Such a scenario would imply that the EDM would be $d_\mu = 2.4(0.4) \times 10^{-19}$ e-cm, a factor $\approx 10^8$ larger than the current limit on the electron EDM. While this would be a very exciting result, it is orders of magnitude larger than that expected from even the most speculative models[47–50]

To reduce systematic errors in the muon EDM measurement, a "frozen spin" technique has been proposed[51] which uses a radial electric field in a muon storage ring, operating at $\gamma << \gamma_{\rm magic}$ to cancel the $(g-2)$ precession. The EDM term, Eq. 19, would then cause the spin to steadily move out of the plane of the storage ring. Electron detectors above and below the storage region would detect a time-dependent up-down asymmetry that increased with time. As in the $(g-2)$ experiments, detectors placed in the plane of the beam would be used, in this case to make sure that the radial-E-field cancels the normal spin precession exactly. Adelmann and Kirsh[52] have proposed that one could reach a sensitivity of $5\times10^{-23}\,e-$cm with a small storage ring at PSI. A letter of intent at J-PARC[53] suggested that one could reach $< 10^{-24}\,e-$cm there. The ultimate sensitivity would need an even more intense muon source, such as a neutrino factory.

5.   The Search for Lepton Flavor Violation

The standard-model gauge bosons do not permit leptons to mix with each other, unlike the quark sector where mixing has been known for many years. Quark mixing was first proposed by Cabibbo,[54] and extended to three generations by Kobayashi and Maskawa,[55] which is described by a mixing 3×3 matrix now universally called the CKM matrix. With the discovery of neutrino mass, we know that lepton flavor violation (LFV) certainly exists in the neutral lepton sector, with the determination of the mixing matrix for the three neutrino flavors having become a world-wide effort.

While the mixing observed in neutrinos does predict some level of charged lepton mixing, it is many orders of magnitude below present experimental limits.[13] New dynamics,[57–66] e.g. supersymmetry, do permit leptons to mix, and the observation of standard-model forbidden processes such as

$$\mu^+ \to e^+\gamma; \quad \mu^+ \to e^+e^+e^-; \quad \mu^-N \to e^-N; \qquad (21)$$

$$\mu^+e^- \to \mu^-e^+; \quad \mu^- + \mathcal{N} \to e^+ + \mathcal{N}' \qquad (22)$$

would clearly signify the presence of new physics. The present limits on lepton flavor violation are shown in Fig. 7.

If lepton mixing occurs via supersymmetry, there will be a mixing between the supersymmetric leptons (sleptons) which would also be described by a 3×3 mixing matrix. The schematic connection between lepton flavor violations and the dipole moments is shown in Fig. 8, and there are models that try to connect these processes.[67]
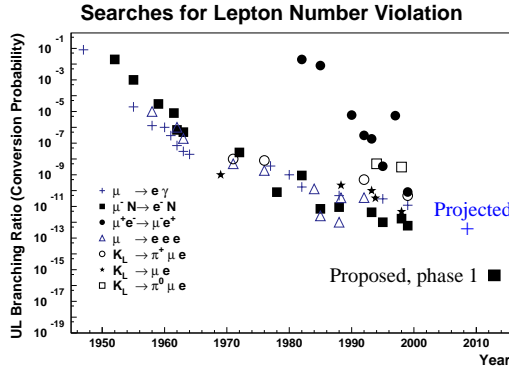
Fig. 7. Historical development of the 90% C.L. upper limits on branching ratios respectively conversion probabilities of muon-number violating processes which involve muons and kaons. Also shown is the projected goal of the MEG ($\mu^+ \to e^+\gamma$) experiment which is underway at PSI, and the projected sensitivity of the recent letter of intent to J-PARC for muon-electron conversion. (Figure from Ref. [13])
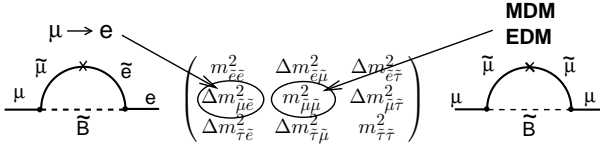

Fig. 8. The supersymmetric contributions to the anomaly, and to $\mu \to e$ conversion, showing the relevant slepton mixing matrix elements. The MDM and EDM give the real and imaginary parts of the matrix element respectively.

In a large class of models, if the $\Delta\ell = 1$ LFV decay goes through the transition magnetic moment, one finds[13]

$$\frac{B(\mu N \to eN)}{B(\mu \to e\gamma)} = 2 \times 10^{-3} B(A, Z), \qquad (23)$$

where $B(A, Z)$ is a coefficient of order 1 for nuclei heavier than aluminum.[68] For other models, these two rates can be the same,[13] so in the design of new experiments, the reach in single event sensitivity for the coherent muon conversion experiments needs to be several orders of magnitude smaller than for $\mu \to e\gamma$ to probe the former class of models with equal sensitivity. Detailed calculations of $\mu - e$ conversion rates as a function of atomic number have also been carried out,[69] and if observed, measurements should be carried out in several nuclei.

From the experimental side, the next generation $\mu \to e\gamma$ experiment, MEG, is now under way at PSI,[70] with a sensitivity goal of $10^{-13} - 10^{-14}$. Since the decay occurs at rest, the photon and positron are back-to-back, and share equally the energy $m_\mu c^2$. This experiment makes use of a unique "COBRA" magnet which produces a constant bending radius for the mono-energetic e^+, independent of its angle. The photon is detected by a large liquid Xe scintillation detector as shown in Fig. 9.

Of the various lepton-flavor violating reactions, only coherent muon conversion does not require coincidence measurements. The decay $\mu \to 3e$, while theoretically appealing, requires a triple coincidence and sensitivity to the whole phase space of the decay, and thereby is ex-
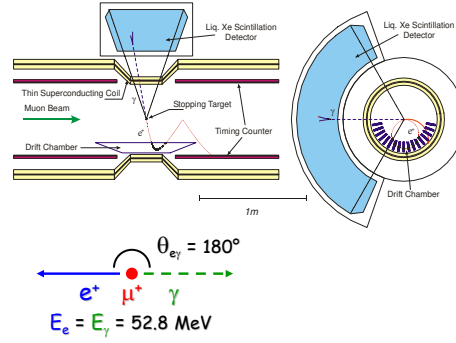

Fig. 9. The side and end views of the MEG experiment. Since the muon is at rest, the photon and positron are at a relative angle of 180°. The positron is tracked in a magnetic field which produces a constant bending radius, independent of angle.

perimentally more challenging. It is the coherent muon to electron conversion, where with adequate energy resolution, the conversion electron can be resolved from background, that with adequate muon flux can be pushed to the 10^{-18} or 10^{-19} sensitivity. Such a program has been proposed for J-PARC.[71]

The muonium to antimuonium conversion (left-hand process in Eq.22) represents a change of two units of lepton number, analogous to $K^0 \bar{K}^0$ oscillations. This process was originally proposed by Pontecorvo.[72] An experiment at PSI[73] obtained a single event sensitivity of $P_{M\bar{M}} = 8.2 \times 10^{-11}$ which implies a coupling $G_{M\bar{M}} \leq 3 \times 10^{-3} G_F$ at 90% C.L., where G_F is the Fermi coupling constant. A broad range of speculative theories such as left-right symmetry, R-parity violating supersymmetry, etc.,[74] could permit such an oscillation.

6. Summary and Conclusions

Since its discovery, the muon has provided an important tool to study the standard model, and to constrain its extensions. Experiments in the planning stage for $(g - 2)$, the search for an electric dipole moment and lepton flavor violation in muon decay or conversion will continue this tradition. Research and development for new more intense muon sources, such as the muon ionization cooling experiment (MICE),[75] will further propel increases in sensitivity. Muon experiments form an important part of the precision frontier in particle physics, which will continue to provide vital information complementary to that from the highest energy colliders.

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